

**100 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.**

**Notice that for the plots and axes, I have set my window to  $x \in [-5, 5], y \in [-4, 4]$**

1. Consider the differential equation  $y'' - 2y' + y = 0$

a. What is the order of the equation?

b. Verify that  $y(x) = Cxe^x$  is a solution of the equation for any  $C \in \mathbb{R}$

c. What is the phrase that describes a DE together with an Initial Condition?

2. Consider the DE  $y' = -\frac{8}{x^3}$ .

a. Find a general solution to the equation

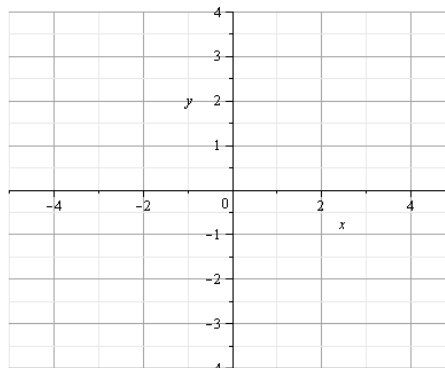
b. Find a particular solution that goes through the point (2,2).

c. Look at the general solution. Explain in a few words why the only linear solution to this equation is vertical.

3. Consider the first-order DE  $y' = -\frac{x}{2y}$ .

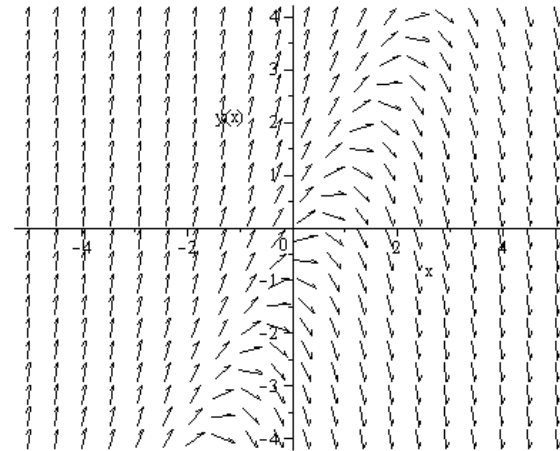
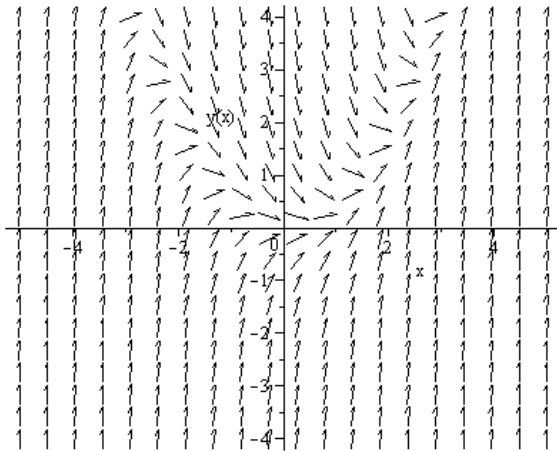
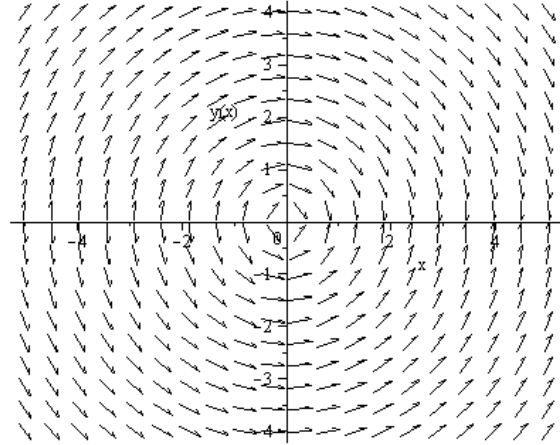
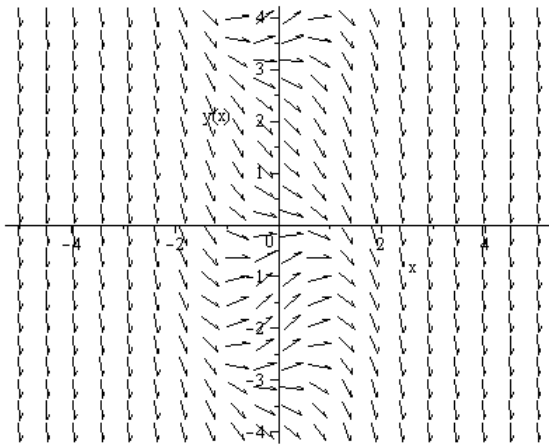
a. Find a general solution

b. Sketch the slope field. This slope field demonstrates why equations such as this cannot be solved explicitly for  $y$  – namely,  $y$  is not strictly a function of  $x$  (it is multi-valued).



4. Match the slope field to the differential equation.

- a.  $y' = -\frac{x}{y}$ . Soln:  $y^2 = -x^2 + C$
- b.  $y' = x^2 - 2y$ . Soln:  $\frac{1}{4} - \frac{1}{2}x + \frac{1}{2}x^2 + Ce^x$
- c.  $y' = y - 2x + 1$ . Soln:  $y = 1 + 2x + Ce^x$
- d.  $y' = -x^2 - \sin y$ . Soln: No solution available



e. For each curve, sketch the solution that corresponds to  $y(2)=1$ .

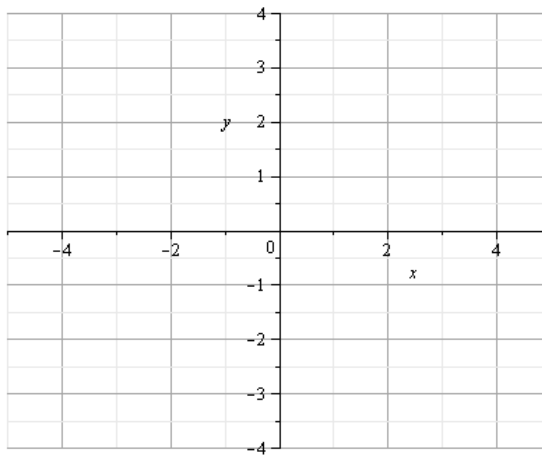
f. One of the equations has a linear solution.

i. Find it and sketch it

ii. What value of C in the general solution generates the linear particular solution?

5. Consider the differential equation  $2x^2y + x^3y' = 1$ .
- a. Find the general solution to the equation

- b. Sketch the slope field for this equation



- c. Find the particular solution satisfying the IC  $y(1) = -1$ , and highlight that solution on the slope field.

6. Find a general solution to the DE  $2xy^2 + x^2y' = y^2$ .

7. Find a general solution  $y(x)$  to the equation  $(y \cos x + 2xe^y)dx + (\sin x + x^2e^y - 1)dy = 0$

8. A projectile is launched straight upward from a height of 30 m and initial velocity 100m/sec.
- Find the maximum height it achieves
  - After how many seconds does it hit the ground?
  - After how many seconds does it pass the launch pad on the way down?
  - After how many seconds is it falling at 60 m/sec?