MAT282 Winter 2010 Exam 3 Apr 6 Name____

1. Write the Differential Equation $t^3x^{(3)} - t^2x'' - 2tx' + x = \ln t$ as a system of first-order equations.

2. Consider the system of differential equations $\begin{Bmatrix} x' = -2y \\ y' = \frac{1}{2}x \end{Bmatrix}$.

   a. Use the method of substitution to find the general solution $\{x(t), y(t)\}$ of the system

   b. Use the identity $A\cos t + B\sin t = C\cos(t-\alpha)$ to show that the solutions of this system are ellipses with axes on the x-axis and y-axis

3. Use the method of elimination to solve the IVP $x_1' = x_1 + 16x_2, x_2' = -2x_1 - 5y; x_1(0) = 1, x_2(0) = 2$.

4. Write the system $x' = 3x - 4y + z + t, y' = x - 3z + t^2, z' = 6y - 7z + t$ in the form $\mathbf{x}' = \mathbf{P}(t)\mathbf{x} + \mathbf{f}(t)$,

where $\mathbf{x} = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}$.

5. Consider the system of first-order equations given by $\begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix}$, where $\mathbf{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

a. Verify that $\mathbf{x_1} = \begin{bmatrix} e^{2t} \\ e^{2t} \end{bmatrix}$ is a solution

b. Verify that $\mathbf{x}_2 = \begin{bmatrix} e^{-2t} \\ 5e^{-2t} \end{bmatrix}$ is a solution.

c. Verify that $\mathbf{x_1}, \mathbf{x_2}$ are linearly independent.

d. Find a particular solution that satisfies $x_1(0) = 5, x_2(0) = -3$

6. Solve each system using the eigenvalue method. If IC are given, find the particular solution. Write your answers in terms of the component functions $\{x_i\}$

   a. $\mathbf{x}' = \begin{bmatrix} -1 & 4 \\ -4 & -9 \end{bmatrix} \mathbf{x}$

   b. $x_1' = x_1 - 2x_2, x_2' = 2x_1 + x_2; x_1(0) = 0, x_2(0) = 4$

c. $x_1' = x_1 + 2x_2 + 2x_3; x_2' = 2x_1 + 7x_2 + x_{3;} x_3' = 2x_1 + x_2 + 7x_3$