MAT282 Winter 2010 Final Exam May 3 Name________________________________

Show your work. You should solve the equations by hand, using your TI only for integration and other computational aspects. **Steps done with your TI should be labeled.**

1. Consider the differential equation $e^y y' = 1$
   a. What is the order of the equation?

   b. Is the equation linear?

   c. Verify that $y(x) = \ln(x + C)$ is a solution of the equation for any $C \in R$

   d. Find the particular solution that satisfies y(0)=0

   e. What is the phrase that describes a DE together with an Initial Condition?

2. Consider the DE $y' - 2xy = 0$.

a. Find the solution using either separation of variables or as a first order linear

b. Find a series solution to the equation, and verify that it matches the analytic solution from part a.

3. For each differential equation, find a general solution. If an initial condition is given, find a particular solution that satisfies the IC.

   a. $xy' - 3y = x^3;\ y(1) = 10$.

   b. $(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0$

c. $y' = y\cos x; y(0) = 1$

d. $y''' + 3y'' - 4y = 0$

e. $y''-2y'+2y=x+1; y(0)=3, y'(0)=1$

4. Consider the system of equations $x' = 3x - y, y' = 5x - 3y$.

   a. Use the method of elimination to find the solution vector $\begin{bmatrix} x \\ y \end{bmatrix}$

   b. Find a particular solution that satisfies the IC $x(0) = 1, y(0) = -1$

5. LaPlace Transforms

   a. Find the LaPlace Transform $L\{(1+t)^3\}$

   b. Find $L^{-1}\left\{\frac{5s-2}{s^2-4s}\right\}$. Use partial fractions.

c. Use the convolution theorem to find $L^{-1}\left\{\dfrac{1}{s(s-2)}\right\}$

6. Consider the system of equations $x_1' = -3x_1 + 4x_2, x_2' = 6x_1 - 5x_2$. Let $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

a. Write the system in the form $\vec{\mathbf{x}}' = \mathbf{P(t)}\vec{\mathbf{x}}$

b. Use the eigenvalue method to find the two solution vectors $\vec{\mathbf{x}}_1(t), \vec{\mathbf{x}}_2(t)$

c. Find the Wronskian $W(\mathbf{x_1}, \mathbf{x_2})$, and explain why it guarantees that the solutions are linearly independent.

d. Write a general solution as a linear combination of $\vec{\mathbf{x}}_1(t), \vec{\mathbf{x}}_2(t)$, and write in the form $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Use c_1, c_2 for your arbitrary constants.

e. Using $c_1 = 2, c_2 = -1$, verify that the resulting particular solution satisfies the equation in the form from part a.

7. Recall the general population equation, $\frac{dP}{dt} = (\beta(t) - \partial(t)) \cdot P(t)$. Assume that the birth □ and death rate □ of a population of fish in a lake are **both inversely proportional to** $\sqrt{P}$. Suppose further that an initial population of 100 fish in a lake grows to 169 after 6 months.

   a. Derive the fact that $P(t) = \left(\frac{t}{2} + 10\right)^2$

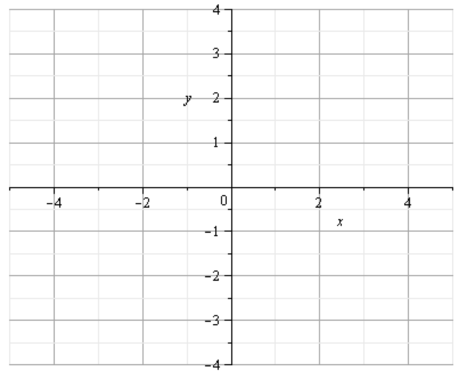
   b. How many fish will there be 1 year after t = 0?

   c. After how many months will there be 500 fish?

8. The brakes are applied to a moving car, causing a constant deceleration of 20 m/sec/sec. The car travels 75 m before coming to a halt. How fast was the car moving when the brakes were applied?

9. Consider the function of period 2π given by $f(t)=\begin{cases} 3; & -\pi < t \le 0 \\ -2; & 0 < t \le \pi \end{cases}$

a. Sketch the function over the primary interval.



b. Find the coefficients for the Fourier series of f(t)