

100 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.

Notice that for the plots and axes, I have set my window to $x \in [-5, 5]$, $y \in [-4, 4]$

1. Consider the differential equation $y'' + y = 3 \cos 2x$

a. What is the order of the equation?

b. Verify that $y(x) = \sin x - \cos 2x$ is a solution of the equation

c. What is the phrase that describes a DE together with an Initial Condition?

2. Consider the DE $y' = -\frac{8}{x^3}$.

a. Find a general solution to the equation

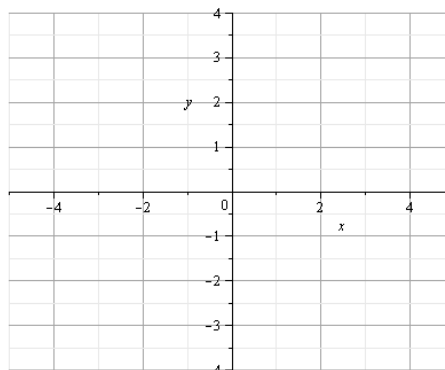
b. Find a particular solution that goes through the point (2,2).

c. Look at the general solution. Explain in a few words why the only linear solution to this equation is vertical.

3. Consider the first-order DE $y' = -\frac{x}{4y}$.

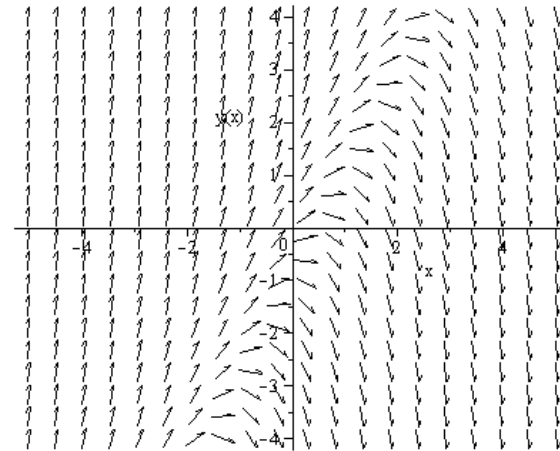
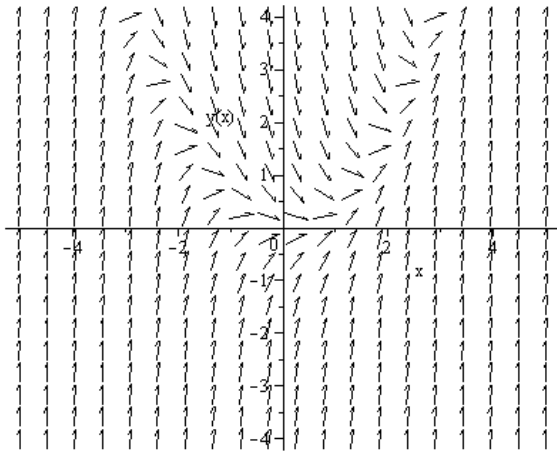
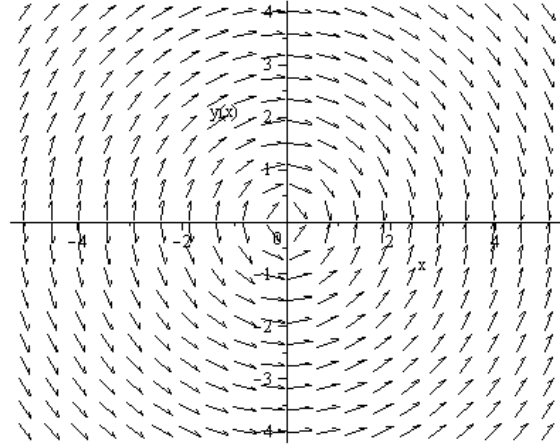
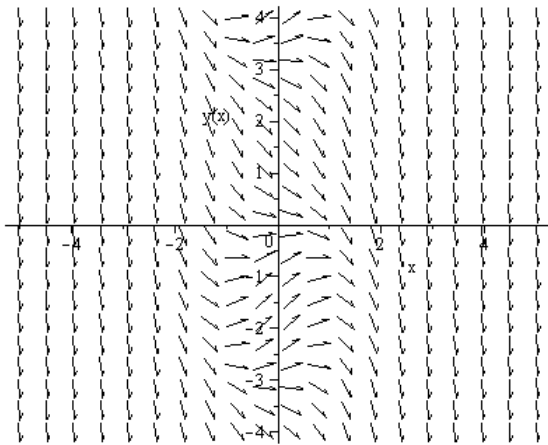
a. Find a general solution

b. Sketch the slope field. This example demonstrates why equations such as this cannot be solved explicitly for y – namely, y is not strictly a function of x (it is multi-valued).



4. Match the slope field to the differential equation.

- a. $y' = -\frac{x}{y}$. Soln: $y^2 = -x^2 + C$
- b. $y' = x^2 - 2y$. Soln: $\frac{1}{4} - \frac{1}{2}x + \frac{1}{2}x^2 + Ce^x$
- c. $y' = y - 2x + 1$. Soln: $y = 1 + 2x + Ce^x$
- d. $y' = -x^2 - \sin y$. Soln: No solution available



e. For each curve, sketch the solution that corresponds to $y(2)=1$.

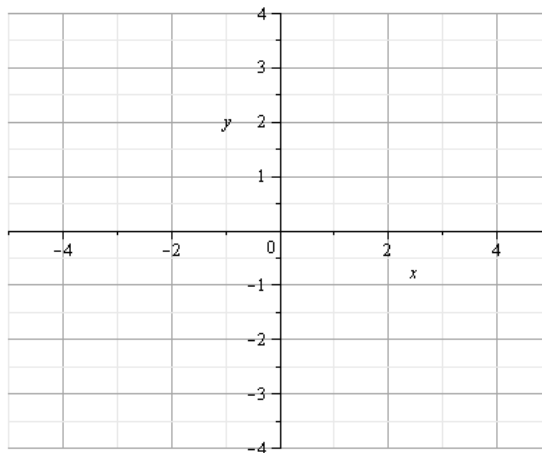
f. One of the equations has a linear solution.

i. Find it and sketch it

ii. What value of C in the general solution generates the linear particular solution?

5. Consider the differential equation $2x^2y + x^3y' = 1$.
- a. Find the general solution to the equation

- b. Sketch the slope field for this equation



- c. Find the particular solution satisfying the IC $y(1)=-1$, and highlight that solution on the slope field.

6. Find the particular solution $y(x)$ to the equation $yy' = x(y^2 + 1)$; $y(0) = 5$

7. A particular strain of bacteria grows at a rate proportional to size of the specimen. At the beginning of an observation 20000 bacteria were present. 10 days later 25000 were present. How many will be present 40 days after the original observation?

8. A projectile is launched straight upward from a height of 30 m and initial velocity 100m/sec.
- Find formulas for the acceleration, velocity, and altitude of the projectile
 - Find the maximum height it achieves
 - After how many seconds does it hit the ground?
 - After how many seconds does it pass the launch pad on the way down?
 - Does it attain a downward velocity of 60 m/sec before hitting the ground?