MAT282 Winter 2011 Exam 2 Mar 4 Name____________________________________

1. (14) Consider the DE $x^2 y''+2xy'-6y=0$.

   a. Verify that $y_1(x)=x^2$ is a solution

   b. A second solution is given by $y_2(x)=x^{-3}$. Compute $W(y_1, y_2)$

   c. Explain briefly why part c. guarantees that y_1, y_2 are linearly independent

   d. Find a particular solution that corresponds to the IC $y(2)=10, y'(2)=15$

2. (16) Consider the IVP $y'' + y' - 2y = 2x; y(0) = 0; y'(0) = 1$.
    a. Find the complementary solution y_c of the associated homogeneous equation.

    b. Find a particular solution $y_p(x)$ by observing the form of f(x) and making an educated guess, the substituting into the original DE and solving.

    c. Form the general solution of the IVP using y_c, y_p

    d. Use y_c, y_p to find solution satisfying the IC

3. (16) Find a general solution to each differential equation
   a. $y^{(3)} - 7y'' + 19y' - 13y = 0$

   b. $y^{(5)} + 3y^{(4)} - 6y^{(3)} - 28y'' - 24y' = 0$

4. (12) Consider the 1st-order IVP $y' = y - x - 1;\ y(0) = 1$, with solution $y_a(x) = 2 + x - e^x$.
   a. Use Euler's method to approximate y(1.0), with step size h = 0.5. Use the last column of the table to store the absolute difference between Euler's approximation to y(x), and the value of $y_a(x)$, using 3 decimal places

i	x_i	Approx y_i	$y_a(x_i)$	Abs err
0	0.00	1.000	1.000	0.000
1				
2				

5. (12) Consider the DE $y^{(3)} + y'' = 3e^x + 4x^2$
   a. Find the roots of the characteristic equation, and form the complementary solution $y_c(x)$

   b. Given the form of f(x), write the initial guess solution $y_i(x)$

   c. Using our rules for removing duplication, combine the answers to parts a and b to write the new trial solution $y_{tr}(x)$. This will have 4 undetermined coefficients in it.

6. (10) Solve the IVP $y''+y=\cos(x); y(0)=1, y'(0)=1$

MAT282 Exam 2

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Problem	Possible Pts	Points
1	14	
2	16	
3	16	
4	12	
5	12	
6	10	
7	20	
Total	100	

Your word problems for exam II are in this packet. The rules for take-home exam are:

1. You may use your book and any notes you have
2. You may ask me for hints
3. You may use your calculator
4. You may **not** discuss the work with your classmates or any other humans

The work is due at class time on March 14. I plan to be in the office on Friday the 11th for questions, or you can e-mail me during the week.

I certify that this exam is the result of my work and mine alone, I have received no help from any sources except those permitted as described in these instructions.

Signed,

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Back of cover sheet/Table of yp's

$f(x)$	y_p
$P_m = b_0 + b_1x + b_2x^2 + \cdots + b_mx^m$	$x^s(A_0 + A_1x + A_2x^2 + \cdots + A_mx^m)$
$a \cos kx + b \sin kx$	$x^s(A \cos kx + B \sin kx)$
$e^{rx}(a \cos kx + b \sin kx)$	$x^s e^{rx}(A \cos kx + B \sin kx)$
$P_m(x)e^{rx}$	$x^s(A_0 + A_1x + A_2x^2 + \cdots + A_mx^m)e^{rx}$
$P_m(x)(a \cos kx + b \sin kx)$	$x^s[(A_0 + A_1x + \cdots + A_mx^m) \cos kx$ $+ (B_0 + B_1x + \cdots + B_mx^m) \sin kx]$

FIGURE 3.5.1. Substitutions in the method of undetermined coefficients.

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Take-home Portion **Due March 14**

7. (20) Solve each applied problem
   a. Suppose that a local water supply is being contaminated slowly. The amount of contaminant is measured in parts per million, the population of which is denoted A(t) and modeled by the equation $\frac{dA}{dt} = kA(t)(200 - A(t))$ where k is a constant. Suppose there are currently 100 ppm of the contaminant in the water, and it is growing at the rate of 1 ppm/yr.
      i. Find a formula for A(t)

      ii. How much of the contaminant will be in the water in 20 years?

      iii. After how many years will the contaminant reach the unsafe level of 180 ppm?

b. A skydiver bails out of an airplane at an altitude of 10,000 feet. She falls for 20 sec, then opens her parachute, then falls the rest of the way to Earth. Both before and after opening the chute, the air resistance is proportional to her velocity – before with proportionality constant $r = .15$, and after with $r = 1.5$.

   i. Find a formula for the velocity during the 20 sec before her chute opened.

   ii. Find her altitude when the chute opens.

   iii. Find her velocity when the chute opens.

   iv. Find a formula for the velocity after the chute opens.

   v. After how many seconds does she reach the ground?