

MAT282 Exam 4

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4/26/2011

Siena Heights University

Problem	Possible Pts	Points
1	14	
2	21	
3	8	
4	7	
Total	50	

This is take-home exam 4:

1. You may use your book and any notes you have
2. You may ask me for hints
3. You may use your calculator
4. You may use your table of LaPlace Transforms
5. You may **not** discuss the work with your classmates or any other humans

The work is due at class time on Apr 26. I will be available for questions on the 25th

I certify that this exam is the result of my work and mine alone, I have received no help from any sources except those permitted as described in these instructions.

Signed,

<u>Function</u>	<u>Laplace</u>
1	$1/s$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}; n \geq 0, s > 0$
$t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$ Note: $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}; \Gamma(x+1) = x\Gamma(x)$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}$
$u_c(t) f(t-c)$	$e^{-cs} F(s) = \frac{F(s)}{e^{cs}} e^{-cs} F(s)$
$e^{ct} f(t)$	$F(s - c)$
$f(ct)$	$1/c F(s/c)$
$(-t)^n f(t)$	$F^{(n)}(s)$ (or $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$)
$f'(t)$	$sF(s) - f(0)$

$$s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

Convolution of product: $L^{-1}\{F(s)G(s)\} = (f * g)(t) = \int_0^t f(x)g(t-x)dx$

1. Find the LaPlace Transform of each function, using the table and linearity:

a. $f(t) = 6e^{-5t} + 5t^3 - 9$

b. $g(t) = e^{3t} + \cos(6t) - e^{3t} \cos(6t)$

2. Find the inverse LaPlace Transform of each function

a.
$$F(s) = \frac{6s}{s^2 + 25} + \frac{3}{s^2 + 25}$$

b.
$$G(s) = \frac{s + 7}{s^2 - 3s - 10}$$

c.
$$H(s) = \frac{2 - 5s}{(s - 6)(s^2 + 11)}$$

4. Solve this IVP using the LaPlace Transform method: $x'' - 10x' + 9x = 5t; x(0) = -1, x'(0) = 2$

5. Use the convolution theorem to find $\mathcal{L}^{-1}\left\{F(s) = \frac{s}{(s+1)(s^2+4)}\right\}$