MAT 282 Winter 2011 Practice Final Exam.

Remember that the final exam is Monday beginning at 11:45

Here is last year's final exam. You can use these problems for practice for the final. I will also likely pull a couple problems directly off of the hour exams – especially ones where you have to identify solution curves. We will discuss late in the week.

During the exam, you may have your yellow-orange LaPlace Transform sheet, your TI NSpire, **and one side of a 3x5 note card containing whatever you like**.

Come to me with questions. I will be at Mathapalooza from 5:00-8:00 Sunday, and in the office beginning at 8:00 Monday, but be aware that **I will not be taking any questions after 11:00, *period***.

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Show your work. You should solve the equations by hand, using your TI only for integration and other computational aspects, and plotting. **Steps done with your TI should be labeled.** Point values are next to the problems.

1. Consider the differential equation $e^y y' = 1$
   a. (2) What is the order of the equation?

   b. (2) Is the equation linear?

   c. (6) Verify that $y(x) = \ln(x + C)$ is a solution of the equation for any $C \in R$

   d. (4) Find the particular solution that satisfies y(0)=0

   e. (2) What is the phrase that describes a DE together with an Initial Condition?

2. (30) Solve **any 3** of these 4 differential equations, If an initial condition is given, find a particular solution that satisfies the IC. In each case y is a function of x.

   a. $xy' - 3y = x^3; y(1) = 10$.

   b. $y' = \frac{1}{\sqrt{x+2}}; y(2) = -1$

c. $y' = y\cos x; y(0) = 1$

d. $y''' + 3y'' - 4y = 0$

3. (12) Solve the IVP $y''-2y'+2y=x+1; y(0)=3, y'(0)=1$

4. (16) Consider the system of equations $x' = 3x - y, y' = 5x - 3y$.

   a. Use the method of elimination to find the solution vector $\begin{bmatrix} x \\ y \end{bmatrix}$

   b. Find a particular solution that satisfies the IC $x(0) = 1, y(0) = -1$

5. (24) LaPlace Transforms
   a. Find the LaPlace Transform $L\{(1+t)^3\}$

   b. Find $L^{-1}\left\{\dfrac{5s-2}{s^2-4s}\right\}$. Use partial fractions.

6. Consider the system of equations $x_1' = -3x_1 + 4x_2, x_2' = 6x_1 - 5x_2$. Let $\vec{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

a. (4) Write the system in the form $\vec{\mathbf{x}}' = \mathbf{P(t)}\vec{\mathbf{x}}$

b. (10) Use the eigenvalue method to find the two solution vectors $\vec{\mathbf{x}}_1(t), \vec{\mathbf{x}}_2(t)$

c. (4) Find the Wronskian $W(\mathbf{x}_1, \mathbf{x}_2)$, and explain why it guarantees that the solutions are linearly independent.

d. (4) Write a general solution as a linear combination of $\vec{\mathbf{x}}_1(t), \vec{\mathbf{x}}_2(t)$, and write in the form $\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Use c_1, c_2 for your arbitrary constants.

e. (6) Using $c_1 = 2, c_2 = -1$, verify that the resulting particular solution satisfies the equation in the form from part a.

7. (12) Use the convolution theorem to find $L^{-1}\left\{\dfrac{1}{s(s-2)}\right\}$

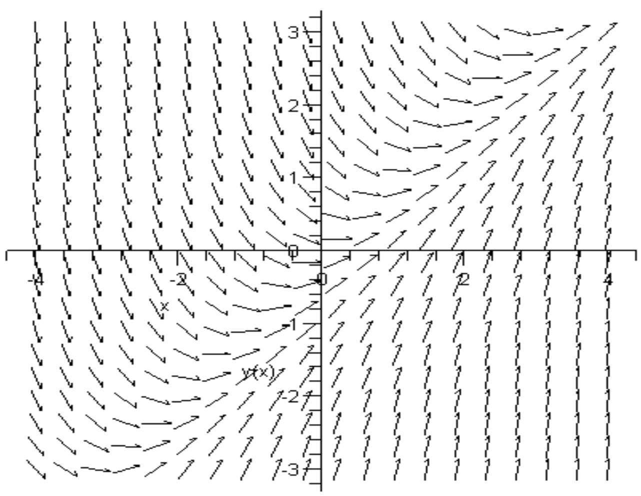
8. (16) You are given 4 DE with their general solutions. Using your TI and a few different values of C, determine which DE/solution pair go with each graph.

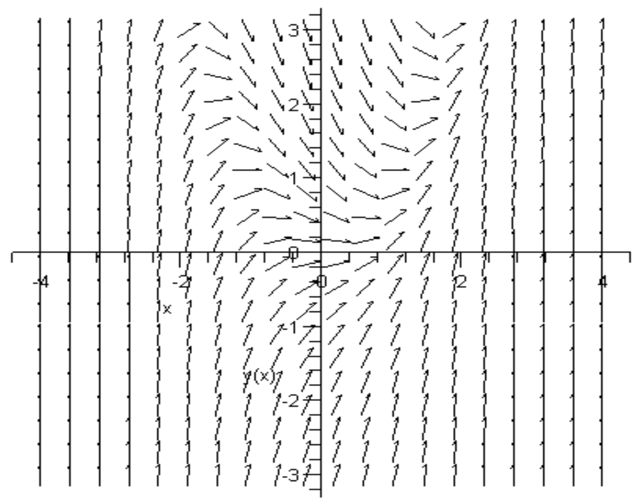
a. $y' = x - y$. Soln: $y = x - 1 + Ce^{-x}$

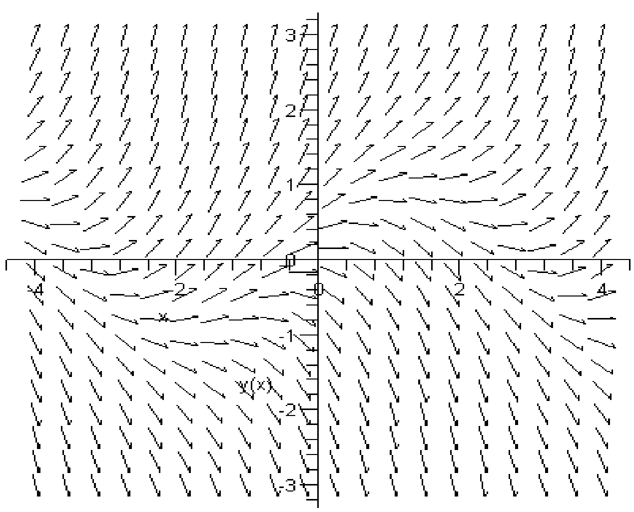
b. $y' = y - x + 1$. Soln: $y = x + Ce^{x}$

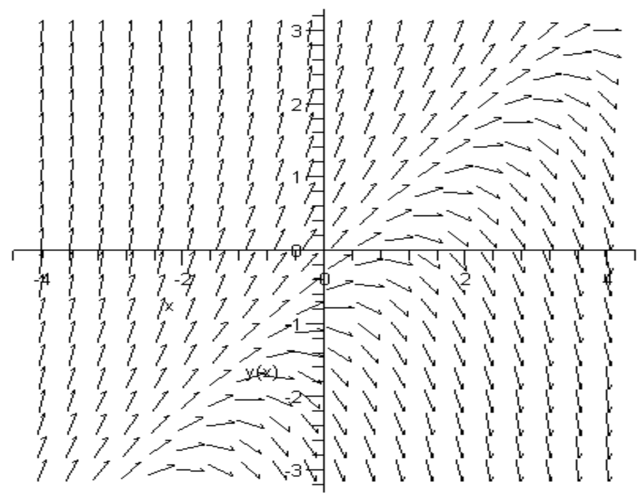
c. $y' = x^2 - y$. Soln: $y = 2 - 2x + x^2 + Ce^{-x}$

d. $y' = y - \sin x$. Soln: $y = \frac{1}{2}\cos x + \frac{1}{2}\sin x + Ce^{x}$









e. In each graph, sketch the solution curve that likely corresponds to the IC y(1) = 1

9. (14) Solve **either one** of the two word problems in this problem.

   a. Recall the general population equation, $\frac{dP}{dt} = (\beta(t) - \partial(t)) \cdot P(t)$. Assume that the birth rate β and death rate δ of a population of fish in a lake are **both inversely proportional to** $\sqrt{P}$. Suppose further that an initial population of 100 fish in a lake grows to 169 after 6 months.

      i. Derive the fact that $P(t) = \left(\frac{t}{2} + 10\right)^2$

      ii. How many fish will there be 1 year after t = 0?

      iii. After how many months will there be 500 fish?

b. The brakes are applied to a moving car, causing a constant deceleration of 20 m/sec/sec. The car travels 75 m before coming to a halt. How fast was the car moving when the brakes were applied?