MAT282 Winter 2014 Exam 1A Feb 05 Name____

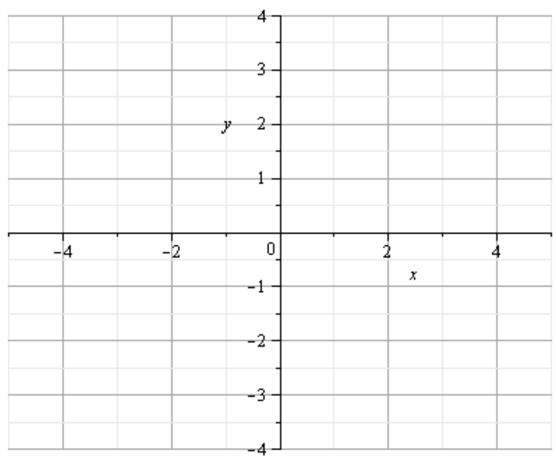
100 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.

1. Consider the differential equation $y''+2y'+1=0$
   a. What is the order of the equation?
   b. Verify that $y(x)=xe^{-x}$ is a solution of the equation
   c. What is the phrase that describes a DE together with an Initial Condition?
2. Consider the separable DE $y'=x^3y^2$.
   a. Find a general solution to the equation
   b. Find a particular solution that goes through the point (1,2).

3. Consider the separable first-order DE $y' = \dfrac{x+1}{y^4+1}$.

   a. Find a general solution

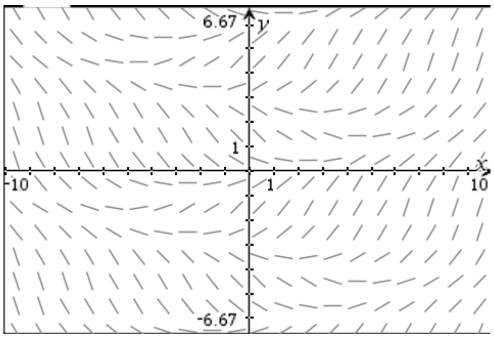
   b. Sketch the slope field, highlighting the particular solution y(1)=0.



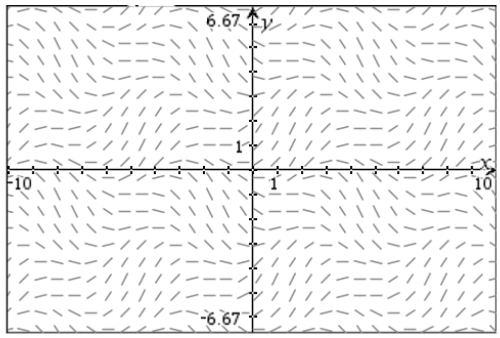
   c. Explain briefly why, looking at the **original DE**, that the solutions near the top and bottom of the grid appear to be (nearly) horizontal

4. Match the slope field to the differential equation. On each curve, sketch the solution that corresponds to y(1)=2. Then give a second IC that generates the same particular solution.

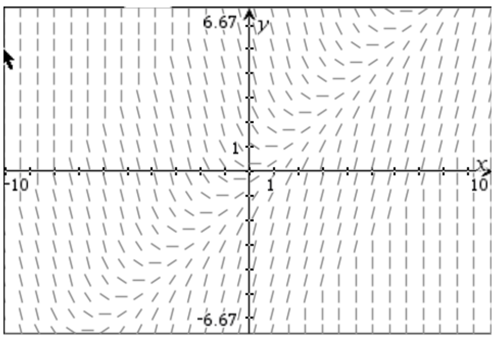
a. $y' = \sin(x) + \sin(y)$ b. $y' = x - y$ c. $y' = 0.2x - \sin y$ d. $y' = x^2 - y - 2$



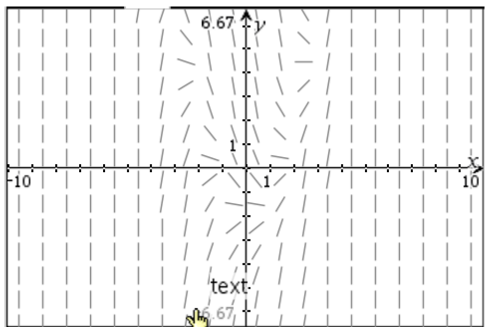
Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________

5. Consider the DE's $xe^{xy}dx + ye^{xy}dy = 0$ and $(y + 2xy^3)dx + (1 + 3x^2y^2 + x)dy = 0$

a. One of these equations is exact. Draw a line through the one that is not. For the one that is exact,

i. Find the general solution

ii. Find the particular solution for y(0) = -3

6. Consider the 1st order linear differential equation $y' + 2xy = xe^{-x^2}$.

   a. Write the complementary homogeneous equation.

   b. Solve the complementary equation and label the solution y_1.

   c. Find the general solution y_g to the equation using variation of parameters.

   d. Find the particular solution $y_p(x)$ that goes through the point (0,3)

7. Find a general solution to each equation (choose your own method). If an IC is given, find the particular solution.

   a. $(2+ye^{xy})+(xe^{xy}-2\text{y})y'=0$

   b. $y'+y=\sin x; y(\pi)=1$

   c. $y'=(64xy)^{\frac{2}{3}}$

8. A projectile is launched straight upward from a height of 100 ft and initial velocity 200 ft/sec. Assume that the acceleration due to gravity is -32 ft/sec/sec
    a. Find formulas for the acceleration, velocity, and altitude of the projectile

    b. Find the maximum height it achieves

    c. Does it attain a downward velocity of 120 ft/sec before hitting the ground?

Formulas which you may or may not use on this exam

a. $c < y < d$. Then the differential equation

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

is exact in R if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

b. For a first order linear differential equation $y' + p(x)y = f(x)$,

  i. To use the integrating factor, $y_g = \frac{1}{\rho} \times \int \rho \cdot f(x)dx$

  ii. To use variation of parameters, $u = \int \frac{f(x)}{y_1(x)}dx$, and $y_g = u \cdot y_1$

c. Motion relationships:

  iii. $v(t) = \int a(t)dt$

  iv. $s(t) = \int v(t)dt$

  v. For vertical projectile motion with gravity the only acceleration, $h(t) = -\frac{g}{2}t^2 + v_0 t + h_0$

d. An equation is ***separable*** if it can be written $y' = \frac{g(x)}{h(y)}$, or $h(y)dy = g(x)dx$