Math 282 Winter 2014 Exam 2 80 pts Feb 20 Name___________________________________

1. Consider the 1st-order IVP $y' = 2 - y; y(0) = 0$.

   a. With step size h = 0.5, use Euler's method (**formula**) to find y_1, the Euler approximation at $x_1 = 0.5$.

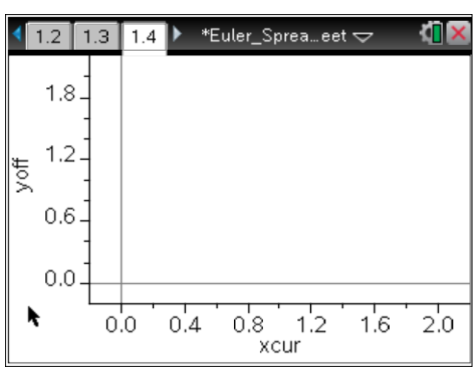
   b. Now use your TI Euler_Spreadsheet file to find the rest of the table values. Place your approximate y's in column 3.

I	x_i	Approx y_i	$y_a(x_i)$
0	0.00	0.000	
1			
2			
3			
4			

   c. Find the analytic solution to the IVP by treating it as a 1st-order linear equation. You may use the integrating factor or Variation Of Parameters.

   d. Fill in column 4 of the table by plugging the x_i's into your answer from part b

   e. On these axes, roughly sketch the Euler approximation (connected segments) and the graph of the analytic solution.



2. Suppose that the half-life of a radioactive substance is 4 days. How long will it take a specimen to decay to $1/100^{th}$ of its original mass?

3. An object is placed in a room where the temperature is 68 ^{o}F. The temperature of the object rises by 9 degrees in 6 minutes and 18 degrees in 12 minutes. Find the temperature of the object when it entered the room.

4. An object weighing 160 lbs. is dropped from a helicopter. The wind resistance during the fall is proportional to the speed at which the object is falling, with constant of proportionality $k = 2.0 \text{ lb/ft}^2$. Find the terminal velocity.

5. A specimen of bacteria exhibit **logistic** growth in a medium with maximum population of M = 50,000. There were 100 bacteria to begin with and 500 one day later.
   a. Find a formula for the number of bacteria after t days.

   b. After how many days will there be 25,000 bacteria, or half of the maximum?

6. Extra Credit. A substance decays at a yearly rate equal to **half of the square of the mass** present. If the original specimen has a mass of 50 g, how long will it be until 25 g remain?

Formulas

Newton's Law Of Cooling: $\frac{dT}{dt} = k(T - T_m)$; $T = T_m + (T_0 - T_m)e^{kt}$

Logistic Growth Model: $\frac{dP}{dt} = k \cdot P \cdot (M - P); P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-Mkt}}$

Exponential Unbounded Growth/Decay: $P'(t) = kP(t); P(t) = P_0 e^{kt}$

Motion: $F = ma; mv' = -mg - kv$. Max Altitude occurs when $t = \frac{\ln\left(\frac{g \cdot m + k \cdot v0}{g \cdot m}\right) \cdot m}{k}$

Gravity only: $h(t) = -\frac{g}{2}t^2 + v_0 t + h_0$

Euler's Method: $y_{n+1} = y_n + f(x, y) \cdot \Delta x$