MAT282 Winter 2014 Final Exam Apr 29 Name____

Show your work. You should solve the equations by hand, using your TI only for integration and other computational aspects, and plotting. **Steps done with your TI should be labeled.** Point values are next to the problems.

1. Consider the differential equation $y' = x - y$
   a. (2) What is the order of the equation?

   b. (2) Is the equation linear?

   c. (6) Verify that $y(x) = Ce^{-x} + x - 1$ is a solution of the equation for any $C \in R$

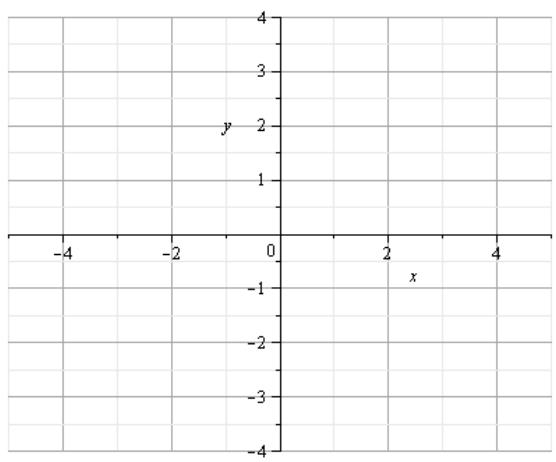
   d. (4) Find the particular solution that satisfies y(0)=10

   e. (2) What is the phrase that describes a DE together with an Initial Condition?

1. Consider the separable first-order DE $y' = \dfrac{x+1}{y^4+1}$.

a. Find a general solution

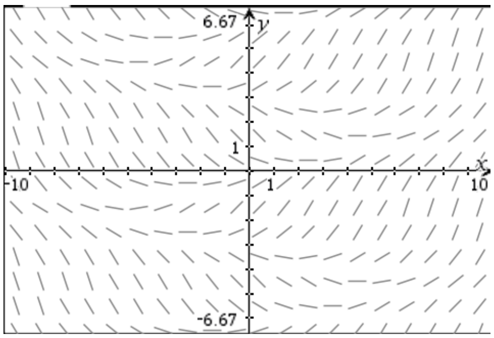
b. Sketch the slope field, highlighting the particular solution y(1)=0.



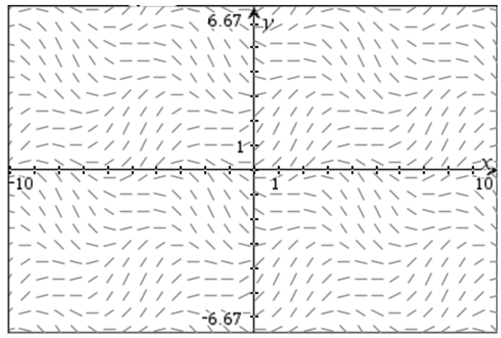
c. Explain briefly why, looking at the **original DE**, that the solutions near the top and bottom of the grid appear to be (nearly) horizontal

3. Match the slope field to the differential equation. On each curve, sketch the solution that corresponds to y(1)=2. Then give a second IC that generates the same particular solution.

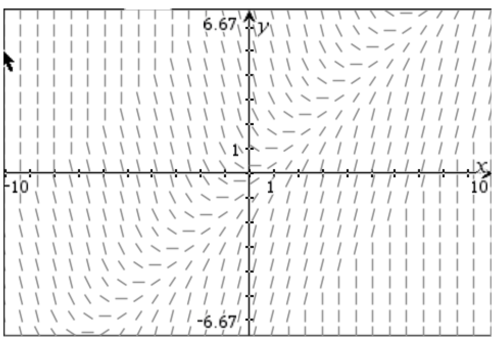
a. $y' = \sin(x) + \sin(y)$ b. $y' = x - y$ c. $y' = 0.2x - \sin y$ d. $y' = x^2 - y - 2$



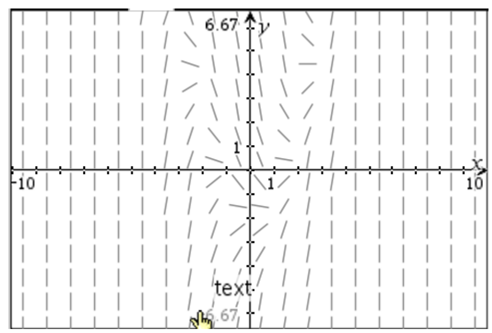
Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________



Eq:_____ Other IC:_____________

4. (24) Solve **any 3** of these 4 differential equations. If an initial condition is given, find a particular solution that satisfies the IC. In each case **y is a function of x**. If you attempt all 4 problems, cross out the one you do **not** want me to grade. Note that there is one Separable, one Exact, one 1st order linear (integrating factor), and one nth order linear homogeneous to choose from.

   a. $xy'+3y=2x^5; y(2)=1$

   b. $\left(2xy^2+3x^2\right)dx+\left(2x^2y+4y^3\right)dy=0$

c. $y' = y\sin x; y\left(\frac{\pi}{2}\right) = 4$

d. $2y''' - 3y'' - 2y' = 0; y(0) = 1, y'(0) = -1, y''(0) = 3$

5. (14) Use undetermined coefficients to solve the IVP $y'' + 9y = 18x$; $y(0) = 1$, $y'(0) = 4$

6. Use LaPlace transforms to solve each IVP. You should use your transform table.

   a. $x''+9x=0; x(0)=3, x'(0)=4$

b. $x''+3x'+2x=t; x(0)=0, x'(0)=2$

7. (14) Consider the system of equations $y_1'=3y_1-y_2, y_2'=5y_1-3y_2$

   a. Use the method of **substitution or elimination** to find the solution vector $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

   b. Find a particular solution that satisfies the IC $y_1(0)=1, y_2(0)=-1$

8. (16) Consider the equation $y' = 2xy$

   a. Find a series solution to the equation

   b. Using the fact that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$, show that your answer from part a is actually $y = c_0 e^{x^2}$

9. (10) Use the characteristic equation to find the general solution to the equation $y''' - y'' + 16y' - 16y = 0$

10. (14) Find the particular solution (only) to the equation $4y''' + 8y'' - y' - 2y = -e^x\left(4 + 45x + 9x^2\right)$

11. (16) Consider the system of equations $y_1' = -3y_1 + 4y_2, y_2' = 6y_1 - 5y_2$. Let $\vec{y} = \begin{bmatrix} y_1(x) \\ y_2(x) \end{bmatrix}$

a. (3) Write the system in the form $\vec{y}' = \mathbf{P}(x)\vec{y}$

b. (10) Use the ***eigenvalue method*** to find the two solution vectors $\vec{y}_1(x), \vec{y}_2(x)$

c. (4) Write a general solution as a linear combination of $\overrightarrow{y_1}(x), \overrightarrow{y_2}(x)$ (in vector form). Use c_1, c_2 for your arbitrary constants.

d. (6) Using $c_1 = 2, c_2 = -1$, verify that the resulting particular solution satisfies the equation in the form from part a.

12. (14) Solve **either one** of the two word problems in this problem.

   a. Assume that the rate of change of a population of fish in a lake is **inversely proportional to** $\sqrt{P}$. Suppose further that an initial population of 100 fish in a lake grows to 169 after **6 months**.

      i. Derive the fact that $P(t)=\left(\frac{t}{2}+10\right)^2$, or $P(t)=\left(6t+10\right)^2$, (depending on your general approach to the problem you will get one of the two of these)

      ii. How many fish will there be **2 years** after t = 0?

      iii. After how many **months** will there be 1000 fish?

b. The brakes are applied to a moving car, causing a constant deceleration of 20 m/sec/sec. The car travels 75 m before coming to a halt. How fast was the car moving when the brakes were applied?