MAT282 Winter 2016 Exam 1 Name________________________________

100 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.

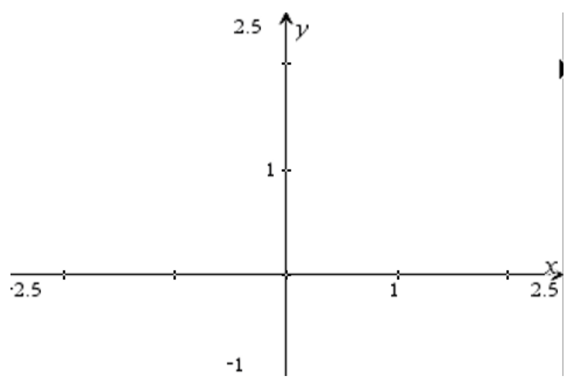
1. Consider the differential equation $y''+2y'+y=0$
   a. What is the order of the equation?
   b. Verify that $y(x)=e^{-x}$ is a solution of the equation
   c. What is the phrase that describes a DE together with an Initial Condition?

2. Consider the separable DE $y'+\frac{3}{x}y=\frac{e^x}{x^3}$.
   a. Use the integrating factor to find a general solution to the equation
   b. Use the general solution to explain briefly why all of the solutions have as an asymptote the negative y-axis

3. Consider the separable first-order DE $y' = x^2\sqrt{y}$.

   a. Find a general solution

   b. Sketch the slope field, setting your window as shown below (two zoom in's at (0,1) will be close).



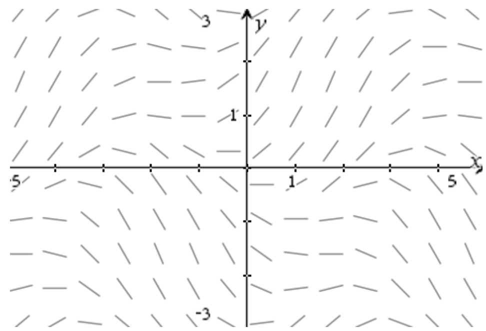
   c. Explain briefly why, looking at the **general solution**, that the solutions are defined only in quadrants I and II.

   d. Add to the graph the solutions that correspond to IC's y(0) = 2, y(0) = 1, and y(0) = 0.

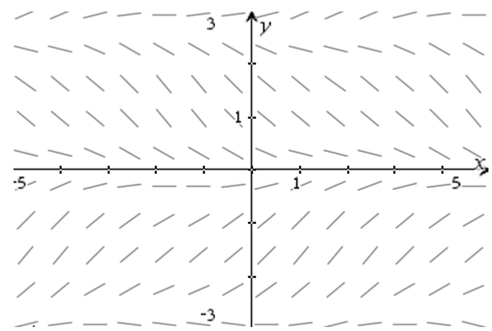
   e. Write a different IC (approximate) that will generate the same solution as y(0) = 2

4. Match the slope field to the differential equation. On each curve, sketch the solution that corresponds to y(1)=-1. Then give a second IC that generates the same particular solution. One "zoom in" at (0,0) will be close to my window settings.

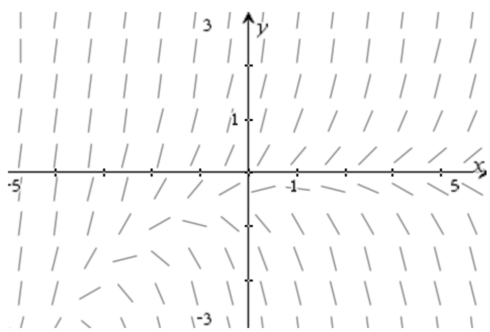
   a. $y' = \sin(x) + \sin(y)$ b. $y' = e^{-\frac{x}{2}} + 2y$ c. $y' = 0.2x - \sin y$ d. $y' = \sqrt{x} - y - 2$



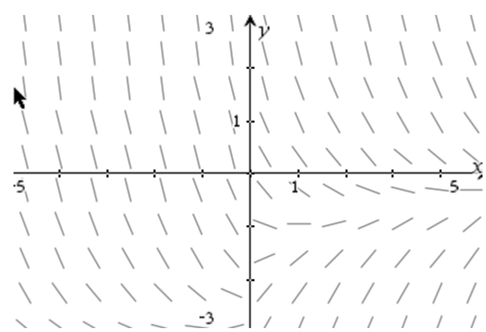
Eq:____ Other IC:___________



Eq:____ Other IC:___________



Eq:____ Other IC:___________



Eq:____ Other IC:___________

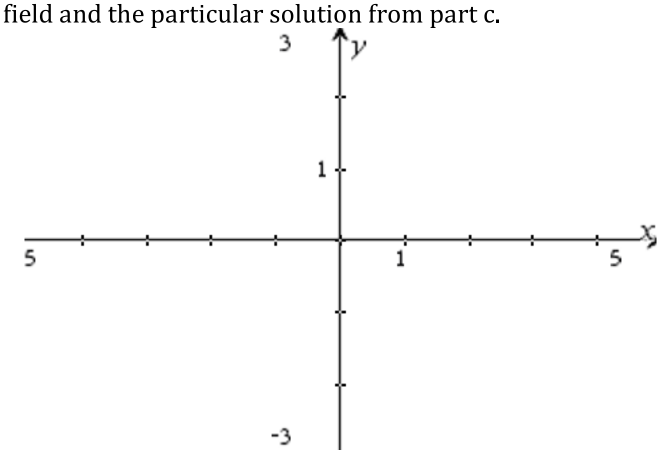
5. Consider the DE $(y + 2xy^3)dx + (1 + 3x^2y^2 + x)dy = 0$

   a. Verify that the equation is exact

   b. Find the general solution

c. Find the particular solution for y(1) = -1

d. Sketch the slope field and the particular solution from part c.



6. Consider the 1st order linear differential equation $y' + 3x^2 y = xe^{-x^3}$.

a. Write the complementary homogeneous equation.

b. Solve the complementary equation and label the solution y_1.

c. Find the general solution y_g to the equation using variation of parameters.

7. For **any two of these three equations**, find a general solution. Choose your own method, **show your work**.

a. $(x+\sin y)+(x\cos y-2y)y'=0$

b. $xy'+y=x\cos x$

c. $y'=\sqrt[3]{8xy}$

8. A missile is launched straight upward from a silo positioned 80 ft below ground level, with initial velocity 500 ft/sec. Assume that the acceleration due to gravity is 32 ft/sec/sec. Use two decimal places where needed.
   a. Find formulas for the acceleration, velocity, and altitude of the projectile

   b. Find the maximum height it achieves

   c. After how many seconds does it leave the silo (cross ground level on the way up)?

   d. What is the velocity when it leaves the silo?

Extra Credit: Prove that any separable equation is exact.

Formulas which you may or may not use on this exam

a. $c < y < d$. Then the differential equation

$$M(x, y)\,dx + N(x, y)\,dy = 0$$

is exact in R if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

b. For a first order linear differential equation $y' + p(x)y = f(x)$,

   i. To use the integrating factor, $y_g = \frac{1}{\rho} \times \int \rho \cdot f(x)dx$

   ii. To use variation of parameters, $y_1 = e^{-\int p(x)dx}, u = \int \frac{f(x)}{y_1(x)}dx$, and $y_g = u \cdot y_1$

c. Motion relationships:

   iii. $v(t) = \int a(t)dt$

   iv. $s(t) = \int v(t)dt$

   v. For vertical projectile motion with gravity the only acceleration, $h(t) = -\frac{g}{2}t^2 + v_0 t + h_0$

d. An equation is ***separable*** if it can be written $y' = \frac{g(x)}{h(y)}$, or $h(y)dy = g(x)dx$