

80 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.

1. Consider the system of differential equations
- $$\begin{aligned}y_1' &= y_1 + 9y_2 \\ y_2' &= -2y_1 - 5y_2\end{aligned}$$
- a. Find the general solution using the method of substitution.

b. Compute the Wronskian $W(y_1, y_2)$

c. Find the particular solution that corresponds to $y_1(0) = 3, y_2(0) = 2$

2. Consider the system of equations
- $$\begin{aligned}y_1' &= 3y_1 - y_2 \\ y_2' &= 5y_1 - 3y_2\end{aligned}$$

a. Write the equation in matrix form

b. Verify that $\bar{y}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$ is a solution

c. Verify that $\bar{y}_2 = \begin{bmatrix} 1 \\ 5 \end{bmatrix} e^{-2t}$ is a solution

d. Form the general solution to the system

e. Find the particular solution that corresponds to $\bar{y}(0) = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$

3. Use the eigenvalue method to find a solution to $\bar{y}' = \begin{bmatrix} 3 & 1 & 1 \\ -5 & -3 & -1 \\ 5 & 5 & 3 \end{bmatrix} \bar{y}$ where $\bar{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

4. Use the eigenvalue method to find a solution to the IVP $\bar{y}' = \begin{bmatrix} -7 & 4 \\ -6 & 7 \end{bmatrix} \bar{y}; \bar{y}(0) = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

5. Use the eigenvalue method to find a solution to $\vec{y}' = \begin{bmatrix} 3 & 2 & -2 \\ -2 & 7 & -2 \\ -10 & 10 & -5 \end{bmatrix} \vec{y}$

6. Consider the differential

a. Write this as a 1st-order system of 2 differential equations.

b. Write your answer from part a. in the matrix form $\bar{y}' = A\bar{y}$

c. Verify that the roots of the characteristic polynomial for the equation in part a. are the eigenvalues of the matrix A in part b.