MAT282 Winter 2016 Final Exam May 04 Name________________________

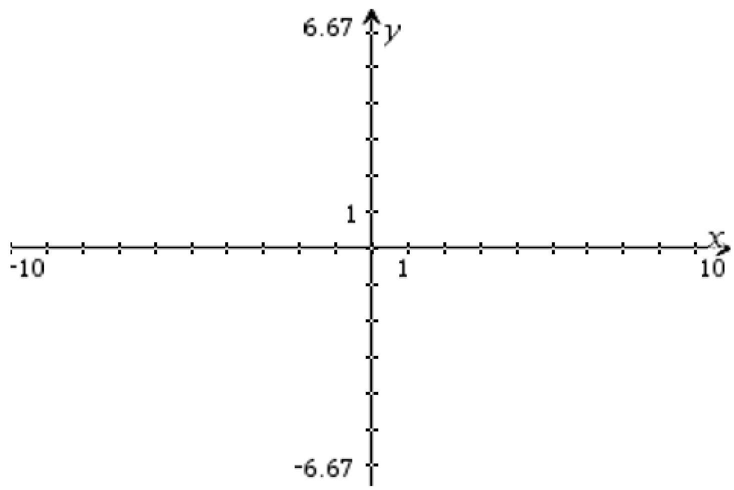
Show your work. Solve the equations by hand, using your TI only for integration and other computational aspects, and plotting. **Steps done with your TI should be labeled.** Point values are next to the problems.

1. Consider the differential equation $y' = x - y$ where y is a function of x
   a. (6) Verify that $y(x) = Ce^{-x} + x - 1$ is a solution of the equation for any $C \in R$

2. For the first order equation $y' = y\sin(x)$
   a. (5) Use separation of variables to find the general solution

   b. (3) Find the particular solution that satisfies $y\left(\frac{\pi}{2}\right) = 2$

c. (4) Sketch the slope field for this equation(rough), and highlight the solution from part b



3. (18) Solve **any 2** of these 4 differential equations. If an initial condition is given, find a particular solution that satisfies the IC. In each case y is a function of x. If you attempt all 4 problems, cross out the one you do **not** want me to grade

   a. $xy'+3y=2x^5;\ y(2)=1$

b. $(2xy^2 + 3x^2)dx + (2x^2y + 4y^3)dy = 0$

c. $2y''' - 3y'' - 2y' = 0; y(0) = 1, y'(0) = -1, y''(0) = 3$

4. (12) Using **undetermined coefficients**, the characteristic equation, and the general form for the right hand side $f(x)$, solve the IVP $y'' + 9y = 18x$; $y(0) = 1$, $y'(0) = 4$

5. (10) Consider the system of equations $\begin{aligned} y_1' &= 3y_1 - y_2 \\ y_2' &= 5y_1 - 3y_2 \end{aligned}$

a. Use the **method of substitution** to find the solution vector $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

b. Find a particular solution that satisfies the IC $y_1(0) = 1, y_2(0) = -1$

6. (18) LaPlace Transforms

a. Use the LaPlace transform method to solve the IVP $y''+3y'+2y=t;\ y(0)=0,\ y'(0)=2$

b. Find $L^{-1}\left\{\dfrac{5s-2}{s^2-4s}\right\}$.

c. Find the LaPlace Transform $L\{(1+t)^3\}$

7. Consider the system of equations $y_1' = -3y_1 + 4y_2, y_2' = 6y_1 - 5y_2$. Let $\vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

a. (3) Write the system in the form $\vec{y}' = P\vec{y}$ (P should have actual numbers in it)

b. (10) Use the ***eigenvalue method*** to find the two solution vectors $\vec{y}_1, \vec{y}_2$

c. (3) Find the Wronskian $W(\vec{y}_1, \vec{y}_2)$, and explain why it guarantees that the solutions are linearly independent.

d. (4) Write a general solution as a linear combination of $\vec{y}_1, \vec{y}_2$. Use c_1, c_2 for your arbitrary constants.

e. (6) Using $c_1 = 2, c_2 = -1$, verify that the resulting particular solution satisfies the equation in the form from part a.

8. (12) Recall the general population equation. Assume that the rate of change of a population of fish in a lake is **inversely proportional to** $\sqrt{P}$. Suppose further that an initial population of 100 fish in a lake grows to 169 after **6 months**.

   i. Derive the fact that $P(t) = \left(\frac{t}{2} + 10\right)^2$, or $P(t) = (6t + 10)^2$ (depending on your general approach to the problem you will get one of the two of these)

   ii. How many fish will there be **2 years** after t = 0?

   iii. After how many **months** will there be 1000 fish?

9. (12) The brakes are applied to a moving car, causing a constant deceleration of 20 m/sec/sec. The car travels 75 m before coming to a halt. How fast was the car moving when the brakes were applied?