MAT282 Winter 2017 Exam 1 Name_____

100 pts. Show your work. If you use your NSpire for a computation, make a note of that fact.

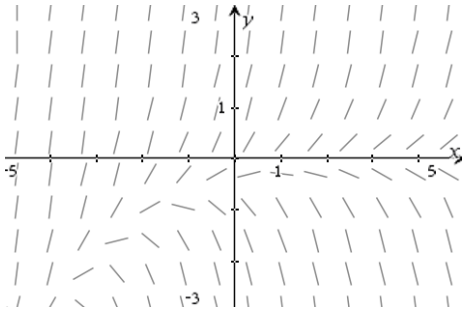
1. Consider the differential equation $y''-4y'+4y=0$
   a. What is the order of the equation?

   b. Verify that $y(x)=e^{2x}$ is a solution of the equation

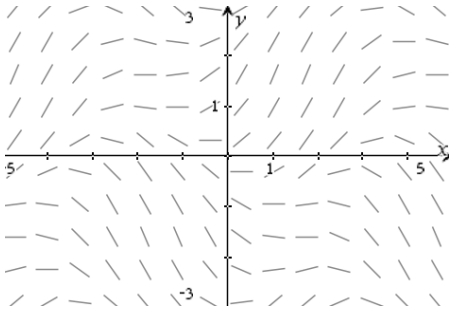
   c. What is the phrase that describes a DE together with an Initial Condition?

2. Match the slope field to the differential equation. On each curve, sketch the solution that corresponds to y(1)=-1. Then give a second IC that generates the same particular solution. One "zoom in" at (0,0) will be close to my window settings.

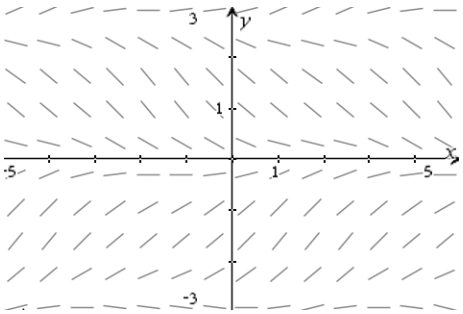
   a. $y'=\sin(x)+\sin(y)$ b. $y'=e^{-\frac{x}{2}}+2y$ c. $y'=0.2x-\sin y$ d. $y'=\sqrt{x}-y-2$



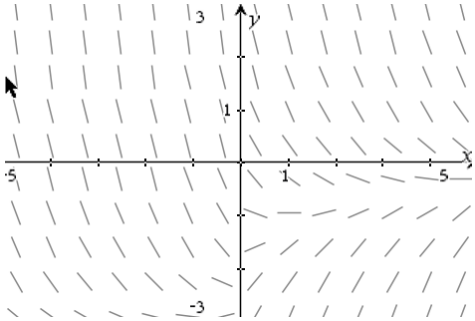
Eq:____ Other IC:_____



Eq:____ Other IC:_____



Eq:____ Other IC:_____

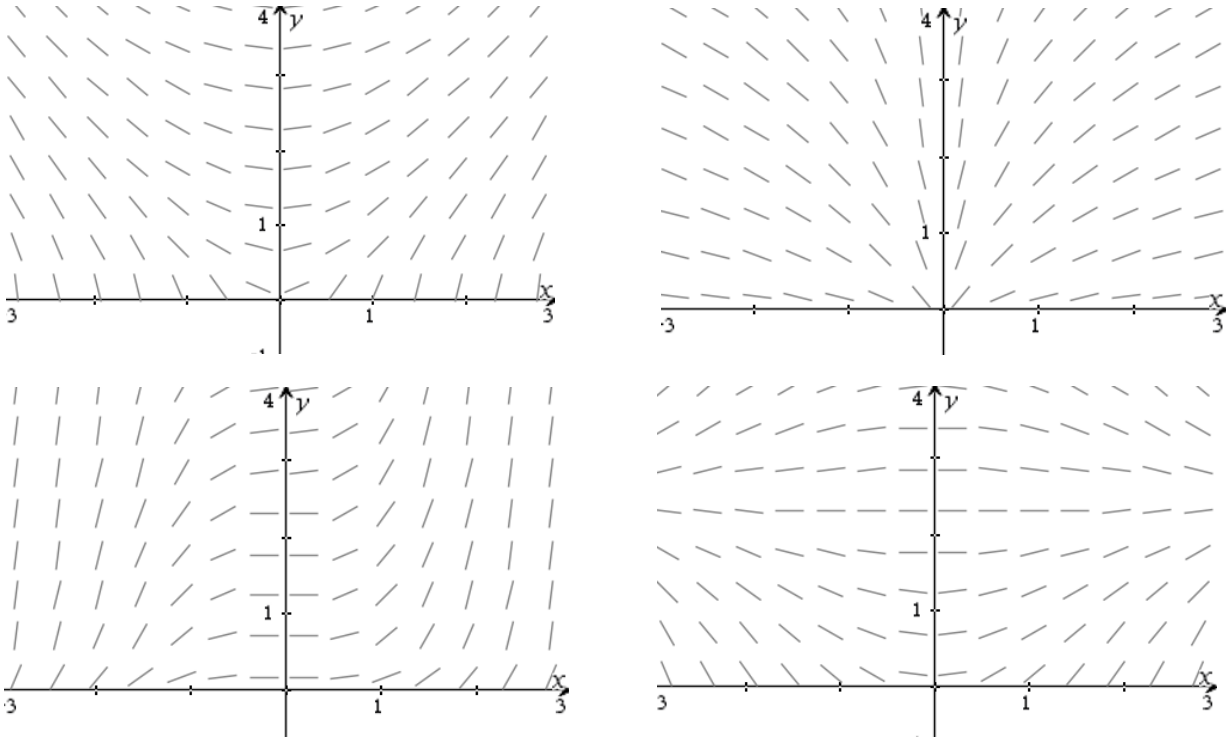


Eq:____ Other IC:_____

3. Consider the **separable** first-order DE $y' = x^2\sqrt{y}$.

   a. Find a general solution

   b. Choose the correct slope field



   c. Explain briefly why, looking at the **general solution**, that the solutions are defined only in quadrants I and II.

   d. **To the graph chosen in part b,** add the solutions that correspond to IC's y(0) = 2, y(0) = 1.

   e. Write a different IC (approximate) that will generate the same solution as y(0) = 2

4. Find the solution to the second order IVP $y'' = 12x + 6; y(0) = 2, y'(0) = -1$

5. Consider the 1st order linear differential equation $y' + 3x^2 y = xe^{-x^3}$.

   a. Write the complementary homogeneous equation.

   b. Solve the complementary equation and label the solution y_1.

   c. Find the general solution y_g to the equation using variation of parameters.

6. A population of bacteria in a Petri dish exhibits logistic growth, with an initial population of 100 bacteria, growth constant k = 0.08, and maximum population of 1000. The time is measured in hours.
    a. Find the formula for the population for any time t.

    b. Find the population after 40 hours.

    c. After how many hours are there 900 bacteria in the dish?

7. A steak is moved from a freezer with temperature -10°C to a refrigerator which is kept at +4°C. 4 hours after it is moved, the steak has risen to a temperature of -5°C. After how many hours will the steak be thawed to +2°C?

8. Solve either one of these two problems

a. $y' + x(y^2 + y) = 0; y(2) = 1$

b. $y' + 2xy = xe^{-x^2}; y(0) = 2$