MAT282 Winter 2017 Final Exam May 02 Name______________________

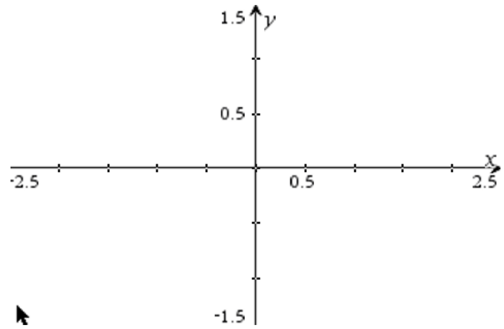
Show your work. Solve the equations by hand, using your TI only for integration and other computational aspects, and plotting. **Steps done with your TI should be labeled.** Point values are next to the problems.

1. (10) Verify that the function $y = e^x \ln x$ is a solution of the equation $y'' - 2y' + y = \frac{1}{x}(y - y')$. Compute the derivatives by hand without your TI.

2. For the first order equation $y' = 6xy^2$.
   a. (8) Use separation of variables to find the general solution. Do all steps by hand.

   b. (4) Find the particular solution that satisfies $y(1) = \frac{1}{5}$

c. (4) Sketch the slope field for this equation, using the TI window as shown, and highlight the solution from part b



3. (20) Find the general solution to each equation.

a. $xy'+2y = x^2 - x + 1; y(1) = \frac{1}{2}$

b. $(3x^2 + 4xy^2)dx + (2y - 3y^2 + 4x^2 y)dy = 0; y(1) = 4$

4. (12) Using **undetermined coefficients**, the characteristic equation, and the form for the right hand side f(x), solve the IVP $y''-4y'-12y=3e^{5x}; y(0)=\frac{18}{7}; y'(0)=-\frac{1}{7}$

5. (20) Solve one of these systems using the method of substitution, the other using the eigenvalue method. The choice is yours

a. $\begin{aligned} y_1' &= 4y_1 - y_2 \\ y_2' &= 2y_1 + y_2 \end{aligned}$; $y_1(0) = 1, y_2(0) = 3$

b. $\begin{aligned} y_1' &= y_1 + 2y_2 \\ y_2' &= 3y_1 + 2y_2 \end{aligned}$; $y_1(0) = 0, y_2(0) = -4$

6. (12) Use the LaPlace transform method to solve the IVP $y''-5y'+6y=te^{2t}; y(0)=0, y'(0)=1$

7. (12) Find each LaPlace transform

a. $L^{-1}\left\{\dfrac{5s^2+6s+4}{(s+4)(s^2+4)}\right\}$

b. $L^{-1}\left\{\dfrac{3s+2}{(s-2)^2+4}\right\}$

8. (6) Find the LaPlace Transform $L\{e^{-7t}t^2+(1+t)^3\}$

9. Consider the 3rd order equation $y''' + 3y'' - 4y = e^{-2x}$

   a. Find the general solution to the corresponding homogeneous equation.

   b. Find a particular solution using undetermined coefficients and the form of f(x).

10. (12) Recall the general population equation. The rate of change of an alligator population P(t) is proportional to P^2. Suppose that the initial population is 12 alligators, and that 10 years later there are 24.

   a. Find a formula for P(t) valid for any t >0.

   b. After how long will there be 48 alligators?

   c. Is the population limited as $t \to \infty$?

11. (12) The rate of decomposition of radioactive einsteinium is proportional to the amount present at any time. The half-life of radioactive einsteinium is 276 days. After 100 days, 0.5 gram remains. What was the initial amount?