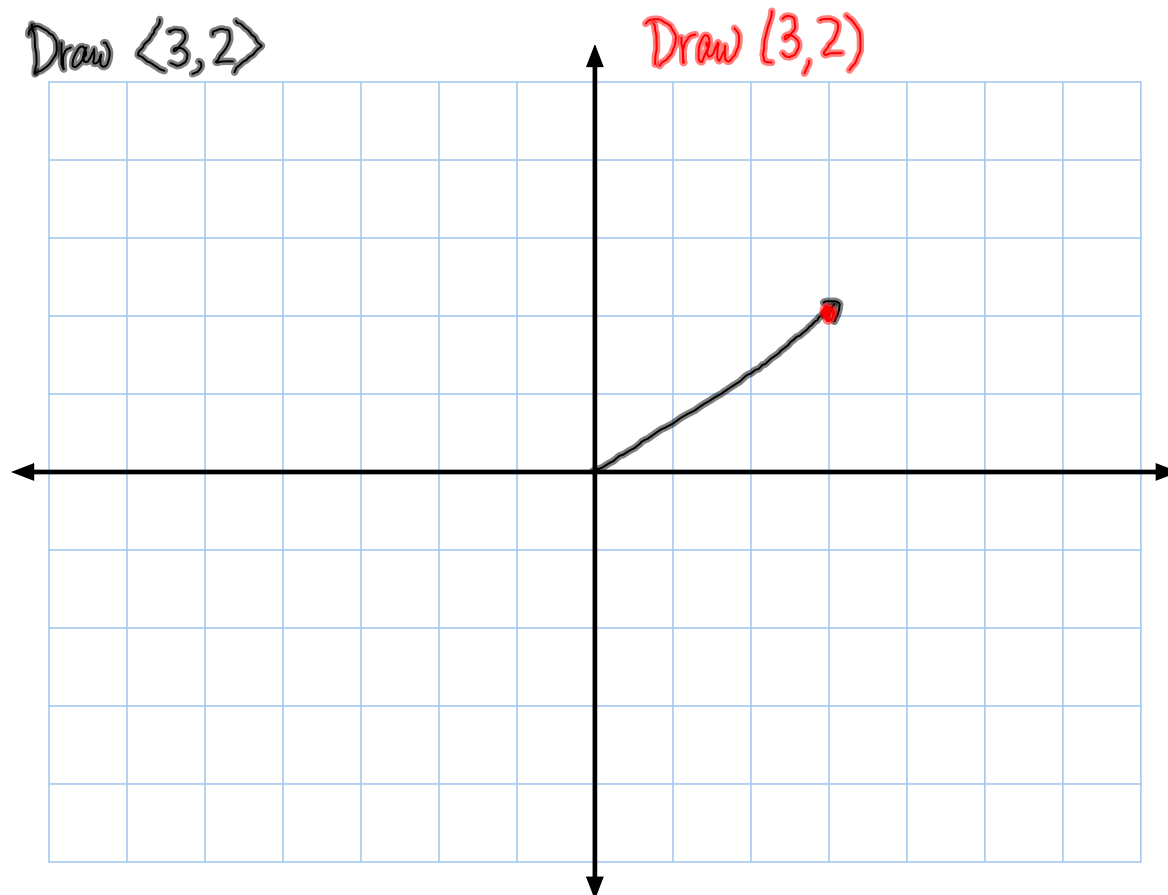


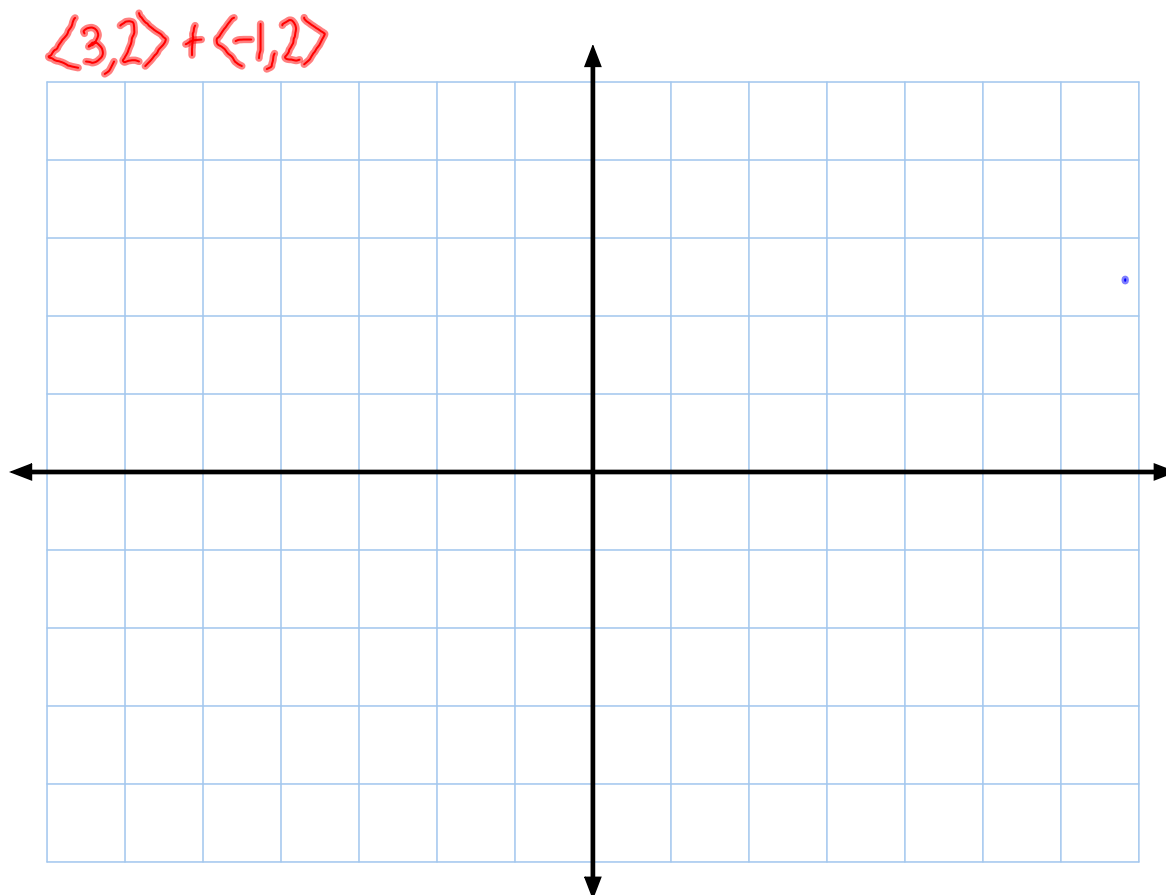
Multivariable Calculus Day 2 - Vectors



Adding Vectors

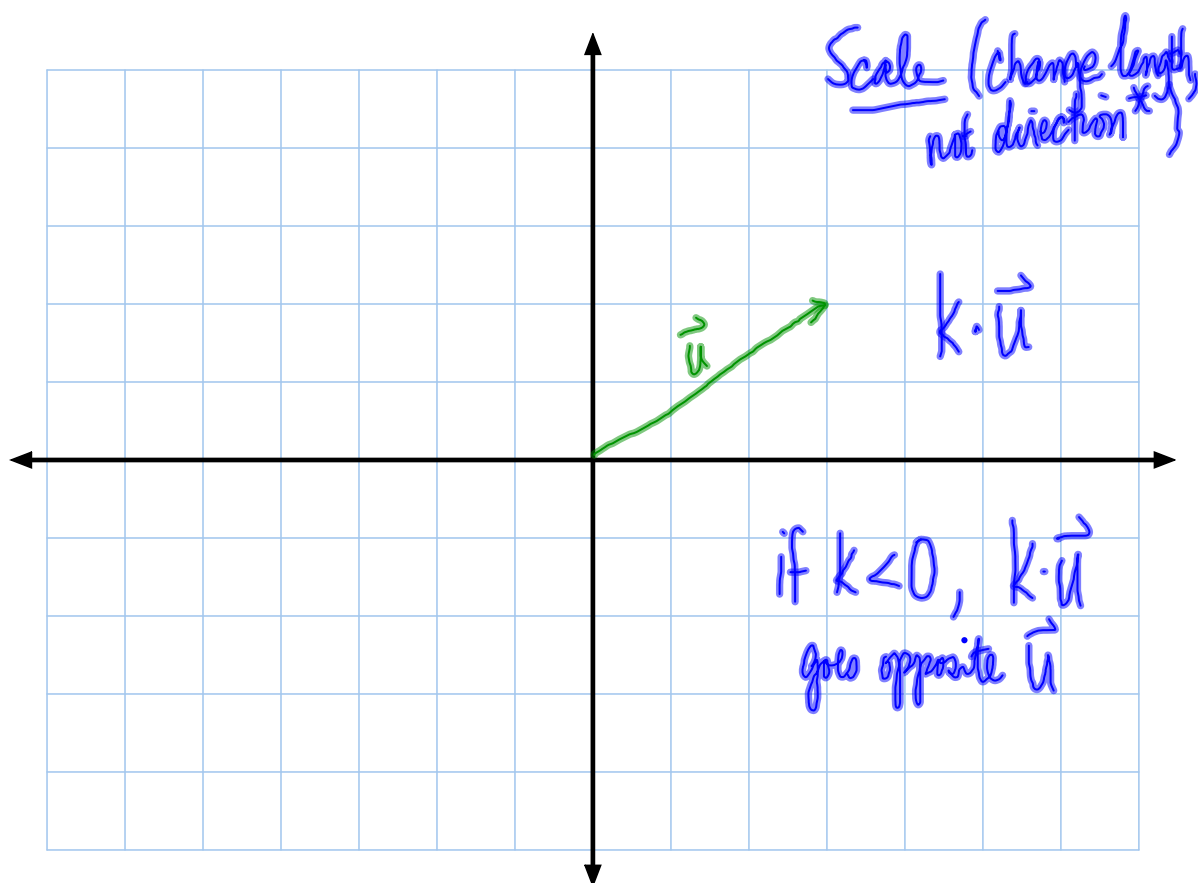
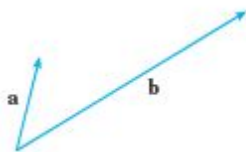
DEFINITION OF VECTOR ADDITION If \mathbf{u} and \mathbf{v} are vectors positioned so the initial point of \mathbf{v} is at the terminal point of \mathbf{u} , then the sum $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of \mathbf{u} to the terminal point of \mathbf{v} .

$$\vec{u}, \vec{v} \quad \vec{u} + \vec{v}$$



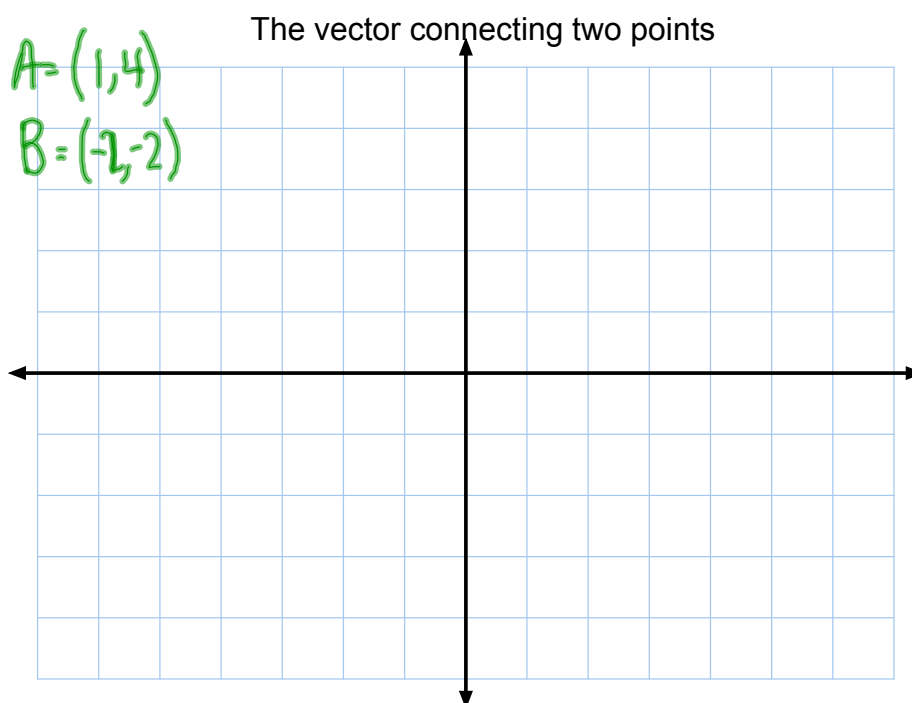
6. Copy the vectors in the figure and use them to draw the following vectors.

- (a) $\mathbf{a} + \mathbf{b}$
- (b) $\mathbf{a} - \mathbf{b}$
- (c) $2\mathbf{a}$
- (d) $-\frac{1}{2}\mathbf{b}$
- (e) $2\mathbf{a} + \mathbf{b}$
- (f) $\mathbf{b} - 3\mathbf{a}$



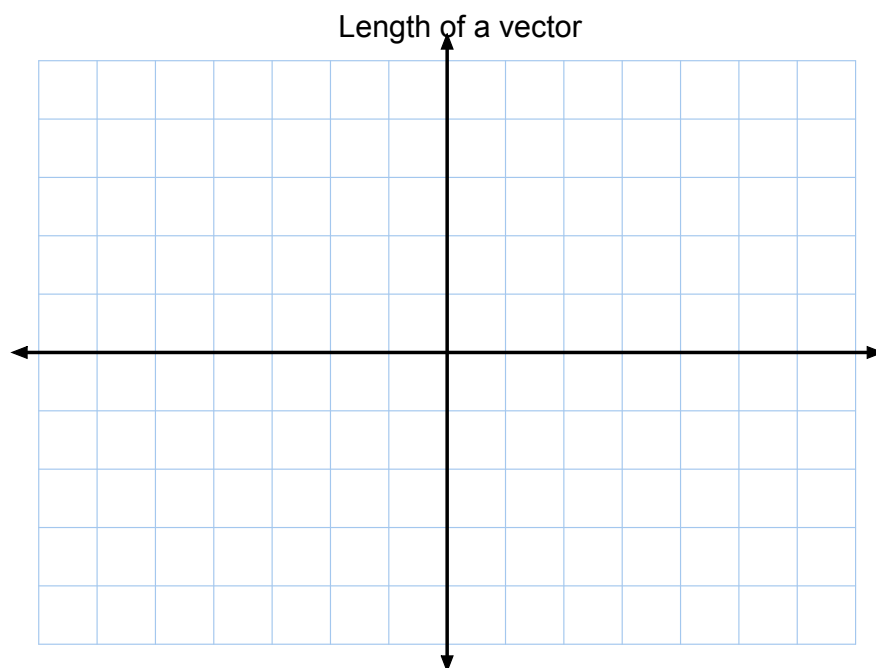
Scaling a vector

DEFINITION OF SCALAR MULTIPLICATION If c is a scalar and \mathbf{v} is a vector, then the scalar multiple $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of \mathbf{v} and whose direction is the same as \mathbf{v} if $c > 0$ and is opposite to \mathbf{v} if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.



I Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector \mathbf{a} with representation \vec{AB} is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$



The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \quad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$

$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarly, for three-dimensional vectors,

$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$

$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$

$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

PROPERTIES OF VECTORS If \mathbf{a} , \mathbf{b} , and \mathbf{c} are vectors in V_n and c and d are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$

2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$

3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$

4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$

5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$

6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$

7. $(cd)\mathbf{a} = c(d\mathbf{a})$

8. $1\mathbf{a} = \mathbf{a}$

$$\mathbf{i} = \langle 1, 0, 0 \rangle \quad \mathbf{j} = \langle 0, 1, 0 \rangle \quad \mathbf{k} = \langle 0, 0, 1 \rangle$$

Standard Basis Vectors

$$\langle 3, -6, 2 \rangle =$$

