Numerical Approximations To Unsolvable Equations.

3.1 - Euler's Method.

Thanks to Mr. Murphy

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There are many differential equations that can not be solved. We can still find an approximate solution that can tell us a lot about the behavior of the model that generated the DE

We will demonstrate with an easy one that can be solved.

Solution:

Suppose we couldn't solve this analytically...

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Euler Method - Extend the "solution" curve using the slope (given)

Remember how it all started? $f'(x) = \lim_{dx \to 0}$

Suppose we took the limit out.
How would that change this? $f'(x) \approx$

for very small values.

Now let's rearrange the terms to

$$f'(x)\,dx \approx f(x+dx) - f(x)$$

We want to find the (approximate) next point. If dx is step size, then
$f(x+dx) \approx$

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This is *Euler's Method of Approximations*

$$f(x+dx) \approx f(x) + f'(x)\,dx$$

We will be given:

1.

2.

3.

At each step we will

-
-
-
-
-

$$f(a+dx) \approx f(a) + f'(a)\,dx$$

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Let’s Demonstrate Euler’s Method On Our Simple Equation.

$$\frac{dy}{dx} = 2x; y(0) = 1; dx = 0.5$$

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$$\frac{dy}{dx} = 2x; y(0) = 1; dx = 0.5$$

Place dot at (0,1)

The next x is 0 + dx = 0 + 0.5 = 0.5

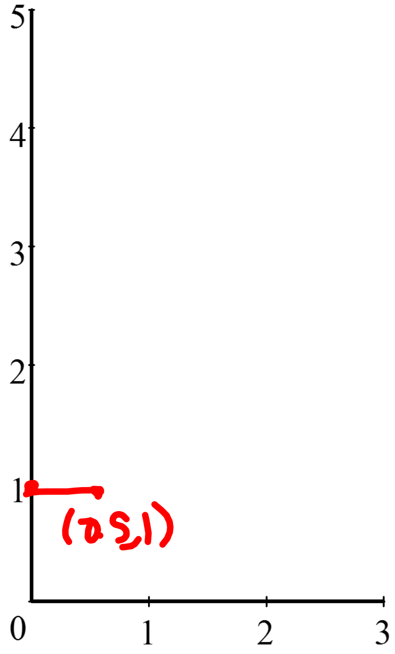
Let's compute the approximate **value** at 0.5

$$f(0+dx) \approx f(0) + f'(0)(0.5)$$

Question: What is $f'(0)$? 2*0 (from the DE)

$$f(0.5) \approx f(0) + 0 \cdot (0.5) = 1 + 0 =$$

So our second point on our approximation is (0.5,1)



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$$\frac{dy}{dx} = 2x; y(0) = 1; dx = 0.5$$

We continue to (1,f(1))

The next x is 0 + dx = 0.5 + 0.5 = 1.0

$$f(a+dx) \approx f(a) + f'(a)dx$$

$$f(0.5+0.5) \approx f(0.5) + f'(0.5)0.5$$

$$f(1) \approx f(0.5) + f'(0.5)0.5$$

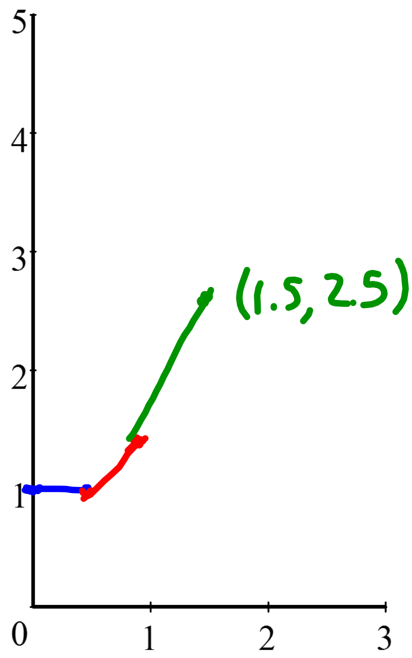
$\rightarrow 2(0.5)$, since $f' = 2x$

$$f(1) \approx 1 + (1)\cdot 0.5$$

$$f(1) \approx 1.5$$

$$f(1.5) \approx ?$$

$$f(2) \approx ?$$



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$$\frac{dy}{dx} = 2x \quad f(0) = 1 \quad dx = 0.5$$

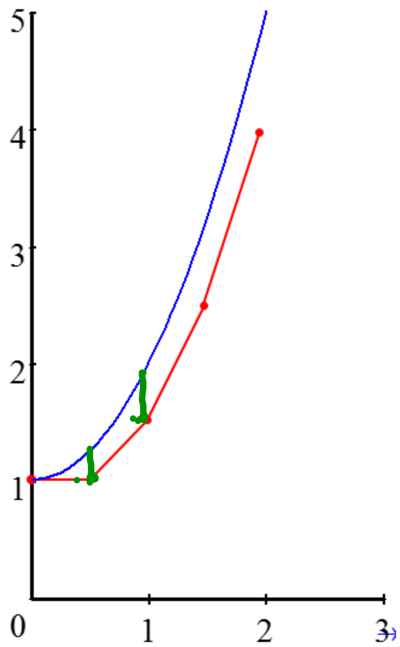
So how would this compare to the real solution?

$$y = x^2 + 1$$

How could we make the approximation better?

Answer:

$dx =$



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Euler's Method steps are often placed in a table like this

Let's try another one that we can solve (1st order linear)

$y' = x + y \qquad y(1) = 2$

$dx = 0.2$

Approximate $y(2)$

Remember:

$$f(a+dx) \approx f(a) + f'(a)\,dx$$

$y' = x + y$

x	y'	y
1		
1.2		
1.4		
1.6		
1.8		
2		

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Solve this equation as a 1st order linear:

$y' - y = x$

$y_1 =$

$u =$

$y_g =$

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